Erratum: Onset of Marangoni convection for evaporating liquids with spherical interfaces and finite boundaries [Phys. Rev. E 84, 046319 (2011)]

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I. INTRODUCTION

In this paper there is an error in Eq. (38), which should have read

$$\left(\int_0^1 -\frac{1}{\sqrt{2}}\frac{\partial \theta_s^L}{\partial \phi}\,dr\right)_{(\phi=\pi/4)} = \left(\int_0^{\pi/4} \frac{\partial \theta_s^L}{\partial r}\sin\phi\,d\phi\right)_{(r=1)}.$$

This correction affects the derivation of the stability parameter, and the changes are described below. There is also an error in Eq. (43), which should have read

$$u_{rs} = \frac{r_I^2\beta}{\rho\alpha}\left(\frac{\partial j_{\rm ev}}{\partial T^L}\theta_s^L + \frac{\partial j_{\rm ev}}{\partial T^V}\theta_s^V\right),$$

and as a result, Eq. (53) should have read

$$u_{rs} = \xi_{CL}\theta_s^L + \xi_{CV}\theta_s^V.$$

The term that was eliminated from Eqs. (43) and (53) did not contribute to the stability analysis, so this change does not affect the derivation, results, or conclusions.

The correction to Eq. (38) affects the derivation of the stability parameter [Eqs. (60)–(77)], so we redo the derivation as follows.

A. Liquid-phase temperature

The general solution to Laplace's equation remains the same. However, the correction to Eq. (38) no longer restricts the modes to $n = 1$. The boundary conditions of Eqs. (38) [corrected] and (41) are satisfied for all modes; therefore, the expression for θ_s^L [Eq. (61)] is now

$$\theta_s^L(r,\phi) = \sum_{n=0}^{\infty}\left(A_n r^n\right) P_n(\cos\phi).$$

This expression will be used in Sec. I D to derive the stability criterion.

B. Vapor-phase temperature

The general solution for the temperature in the vapor phase remains the same. However, the correction results in a change when substituting into Eq. (55) that yields

$$\sum_{n=0}^{\infty}\{D_n[\xi_{TV} - K(n+1)]\}P_n(\cos\phi) = \sum_{n=0}^{\infty}[A_n(n+\xi_{TL})]P_n(\cos\phi).$$

Therefore $D_n = A_n(n+\xi_{TL})/[\xi_{TV} - K(n+1)]$, and the expression for θ_s^V [Eq. (64)] is

$$\theta_s^V(r,\phi) = \sum_{n=0}^{\infty}\frac{A_n(n+\xi_{TL})}{[\xi_{TV} - K(n+1)]}r^{-n-1}P_n(\cos\phi).$$

Similar to the expression for the liquid-phase temperature, this expression will be used in Sec. I D to derive the stability criterion.

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C. Liquid-phase radial velocity

The general solution to the spherical biharmonic equation remains the same, and the boundary condition from Eq. (39) is still satisfied. However, substituting Eq. (65) into Eq. (53) now yields

$$G_0 + G_1 \cos\phi + \sum_{n=0}^{\infty} (E_n + F_n) P_n(\cos\phi) = \sum_{n=0}^{\infty} A_n \left[\xi_{CL} + \frac{\xi_{CV}(n+\xi_{TL})}{\xi_{TV} - K(n+1)}\right] P_n(\cos\phi). \tag{1}$$

The expression for u_{rs} [Eq. (68)] is now

$$u_{rs}(r,\phi) = G_0 r + G_1 \cos\phi + \sum_{n=0}^{\infty} \left(E_n r^{n+2} + F_n r^n\right) P_n(\cos\phi).$$

This expression will also be used in Sec. I D to derive the stability criterion.

D. Examination of the coefficients

Equation (69) remains the same; however, substitution of the solutions θ_s^L, θ_s^V, and u_{rs} in their corrected form from above yields

$$\xi_{Ms} = \frac{1}{A_n}\left[\frac{-2E_n(n+1)(n+2) - 2F_n(n^2+n-1)}{n(n+1)}\right]. \tag{2}$$

The E_n and F_n coefficients in Eq. (2) are investigated individually. There are two possible cases: Eq. (1) can be solved either for E_n with F_n set equal to zero (Case E) or for F_n with E_n set equal to zero (Case F). Utilizing combinations of these terms would result in a description of the stability parameter with ambiguous constants, which would have to be eliminated, so only these two cases are considered. Case E yields

$$\xi_{Ms}^E = -\left[\frac{2(n+2)}{n}\right]\left[\xi_{CL} + \frac{\xi_{CV}(n+\xi_{TL})}{\xi_{TV} - K(n+1)}\right], \tag{3}$$

and Case F yields

$$\xi_{Ms}^F = -\left[\frac{2\left(n^2+n-1\right)}{n(n+1)}\right]\left[\xi_{CL} + \frac{\xi_{CV}(n+\xi_{TL})}{\xi_{TV} - K(n+1)}\right]. \tag{4}$$

The form given in Eq. (4) is a multiple of Eq. (3), so only Eq. (3) needs to be analyzed since the multiplier is larger for all values of n greater than zero, and therefore the parameter is larger and corresponds to the least stable case.

E. Stability parameter for a conducting funnel wall

The correction from above has generated Eq. (3) to replace Eq. (71) from the paper. The simplifications and rearrangement of this equation follows the same procedure in the paper, and the result is that Eq. (77) is replaced with a more general stability parameter:

$$\chi_s = \frac{\nu}{\gamma_T}\left\{\left.\frac{\partial j_{\text{ev}}}{\partial T^L}\right|_I + \left.\frac{\partial j_{\text{ev}}}{\partial T^V}\right|_I \left[\frac{n+\xi_{TL}}{\xi_{TV} - K(n+1)}\right]\right\}. \tag{5}$$

II. EXPERIMENTAL RESULTS

In the experimental observations presented in the paper, only the $n = 1$ mode is present. With the error noted above in Eq. (38) the system was mathematically restricted to $n = 1$, so we had no need to demonstrate the presence of only the $n = 1$ mode. However, with the correction we must now show explicitly that the $n = 1$ mode is the mode associated with the Marangoni convection.

Temperature measurements made along the interface are summarized in Fig. 1, which is reproduced from Ref. [2] in the paper. It can be seen that during the quiescent evaporation in experiment EV5 the temperature along the interface was uniform, within the measurement error bars. In contrast, for evaporation with Marangoni convection present, experiment EV19 shows a continuous increase in temperature from the apex to the contact line. This is indicative of one large circulation cell present in the bulk liquid phase, since the presence of many cells would require temperature fluctuations along the interface caused by the hot and cold regions in the ascending and descending regions of each circulation cell. Therefore, it is concluded that there is one large circulation cell in the experimental observations. For the spherical harmonics, the presence of one circulation cell indicates that only the $n = 1$ mode is present, because higher values of n would result in the presence of additional cells. The general stability parameter from Eq. (5) above can be simplified for these experiments, so we recover Eq. (77) from the paper:

$$\chi_s^{(n=1)} = \frac{\nu}{\gamma_T}\left[\left.\frac{\partial j_{\text{ev}}}{\partial T^L}\right|_I + \left.\frac{\partial j_{\text{ev}}}{\partial T^V}\right|_I \left(\frac{1+\xi_{TL}}{\xi_{TV} - 2K}\right)\right], \tag{6}$$

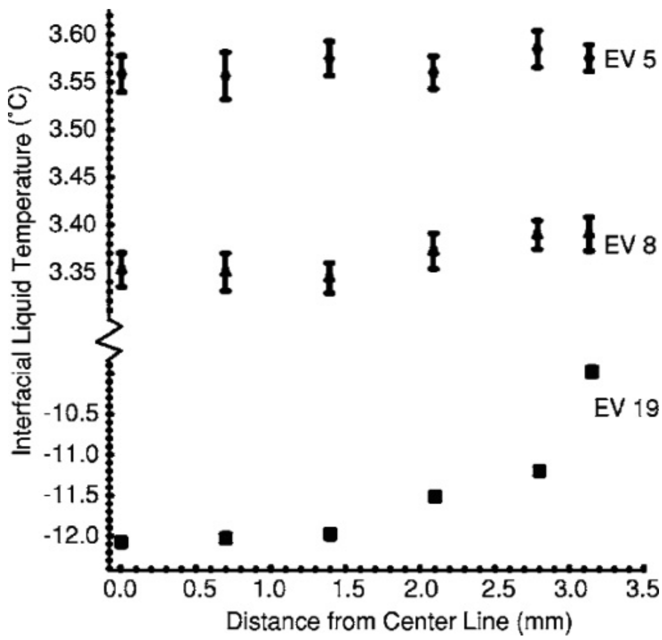

FIG. 1. The interfacial liquid temperature measured as a function of distance from the center line of the funnel in three different experiments. Experiment EV19 is one for which Marangoni convection is present. (Reproduced from Ref. [2] in the paper.)

and the onset is predicted to occur for a value of 1/6. Therefore, the analysis from Sec. VII onward remains unchanged, and the results and conclusions are unchanged.